3. Discrete and Continuous-Time Analysis of Current-Mode Cell

3.1 Introduction

Fig. 3.1 shows schematics of the basic two-state PWM converters operating with current-mode control. The sensed current waveform is added to an external ramp, and the peak (or valley) of the waveform is compared to a control signal to turn off (or turn on) the power switch. For the purpose of this chapter, perturbations of input and output voltage will not be considered, and only the current-mode portion of the circuit is analyzed. Fig. 3.2 shows the basic converters with the input and output voltages represented by fixed sources. All of these converters have a commonality.
When the switch is turned on, the dc voltage \( V_{on} \) is applied across the inductor. When the switch is turned off, the dc voltage \( V_{off} \) is applied across the inductor. The generic current-mode cell, shown in Fig. 3.3, therefore represents all of the converters with current-mode control. For the buck-boost converter only, the input and output voltages are equal to the on-time voltage and off-time voltages, respectively. In general, the on-time and off-time voltages are linear combinations of input and output voltages to the current-mode cell.

Analysis of this reduced block is analogous to the analysis of the PWM switch block where only the nonlinear elements of the circuit are extracted and replaced with their equivalent small-signal model. Sampled-data analysis will be used for the analysis of the current-mode block, and the results will provide a model which can be inserted into the full converter. This will be done in a later chapter, and feedforward terms will be introduced to complete the small-signal model.

It has been shown that the switch model provides accurate power stage transfer functions up to half the switching frequency. Referring to Fig. 3.1, it is apparent that the structure of the basic single-loop converter still exists when current-mode control is used. The converter is still controlled by a duty cycle input, \( d \), and still produces average outputs from the states. The role of the switch model and modulator gain remain unchanged with current-mode control. The fundamental difference from average control methods, where switching frequencies are removed by filtering, is that current-mode control uses an instantaneous value of
the inductor current. This introduces phenomena unique to current-mode control which should be accounted for by a revised model of the current feedback.

Fig. 3.4 shows the structure of the small-signal model for the current-mode cell. The modulator gain, $F_m$, PWM switch model, and linear feedback gain, $R_i$, are the same as they would be for any average control methods. Transfer functions can be experimentally verified for these portions of the model. A gain term, $H_e(s)$, is included in the feedback loop of the inductor current. This block will be used to provide the accurate model for current-mode control where the instantaneous value of current is used for control. Another gain block, $F_c$, is in series with the control voltage to provide flexibility for the model to represent different modulation schemes. For constant-frequency modulation, this gain is unity. The voltage sources of Fig. 3.3 become short circuits in the small-signal model since these sources are fixed.

The purpose of this chapter is to find the form of $H_e(s)$ and $F_c$ for constant-frequency, constant on-time, and constant off-time control. The gain $H_e(s)$ will first be found indirectly by deriving the sampled-data expression for control-voltage-to-inductor-current with the current loop closed, for constant-frequency control. All quantities in the circuit are known except $H_e(s)$, which can then be solved for. It will then be shown that the simple form of $H_e(s)$ can be derived directly from a discrete-time system representing the modulator feedback.
Constant on-time and off-time control systems have an added complexity of a modulator gain with frequency-dependent phase characteristics. The model for these control systems will be derived by showing their similarity to constant frequency control with the appropriate external ramp in the modulator.

### 3.2 Discrete-Time Analysis of Closed-Loop Controller

Fig. 3.5 shows the operation of the constant-frequency, current-mode controller, with the clock initiating the on-time, and the sampled control signal ending the on-time. The sampling instant for the system is at the end of the on-time since this is when the control signal, $v_c$, is used. The current-ripple is not assumed to be small, but the constant input and output voltages of the current-mode cell ensure that the slopes of the current are constant. (Linear ripple.)

Fig. 3.5 shows the effect of a small perturbation $i_L(k)$ occurring at time $t = k$, assuming that other input perturbations to the system are zero. This gives the natural response of the converter. The difference in the steady-state waveform and the perturbed waveform gives the exact small-signal perturbation shown in Fig. 3.5b. Notice that in this waveform, the sampling instant is not constant, but it is shifted by a small amount each time the current intersects the control reference. However, this small-signal perturbation can be approximated with insignificant loss of accuracy by the waveform of Fig. 3.5c. Notice that this final
Figure 3.1. **PWM Converters with Current-Mode Control:** The instantaneous value of inductor current is summed with an external ramp, and used to control the turn-on or turn-off of the switch.
Figure 3.2. Current-Mode Converters with Fixed Input and Output Voltages: The accurate current-mode analysis will be performed on the current-mode control converters with fixed voltages at the input and output. In a later chapter, perturbations in these voltages will be modeled to complete the analysis.
Figure 3.3. **Generic Current-Mode Cell:** The generic cell represents all of the converters with current-mode feedback. The input and output voltages are now the quantities $V_{on}$ and $V_{off}$, which are combinations of the input and output voltages of the different PWM converters.
Figure 3.4. Small-Signal Model of the Current-Mode Cell with Fixed Voltages:
Gain blocks $H_c(s)$ and $F_m$ will be used to model all of the phenomena observed for the current-mode cell of Fig. 3.3. Other quantities in the figure remain the same as predicted by average analysis.
The inductor-current waveform is controlled by a fixed reference, $V_e$, summed with an external ramp, $S_e$. Steady-state waveforms are shown with solid lines. A perturbation $\hat{i}_L(k)$ is introduced at time $t = k$, and the dashed lines show the propagation of the disturbance over subsequent cycles.

Fig. b shows the difference between the steady-state and the perturbed waveforms, giving the small-signal perturbation.

Fig. c shows the approximate small-signal perturbation to a pure discrete-time system.
waveform has the characteristics of a familiar first-order sample-and-hold system, with a constant sampling interval, $T_s$. (The original waveform had a sampling period of $t_s = T_s + \hat{t}_s$, and the perturbation in switching times produces products of small-signal terms which can be ignored.) This is discussed in more detail in [38]. The perturbation introduced at time $t = k$ is held constant until the next sampling instant. The difference between the exact waveform and the approximate waveform is the finite slope of the exact waveform. For small-signal perturbations, this difference is insignificant.

The first step in analyzing such a system is to derive the discrete-time equation describing the change in inductor current from one sampling instant to the next. In the discrete-time domain, the natural response of the approximate waveform of Fig. 3.5c is given by

$$\hat{i}_L(k + 1) = -\alpha \hat{i}_L(k) \quad (3.1)$$

where, with the clock initiating the on-time,

$$\alpha = \frac{S_f - S_e}{S_n + S_e} \quad (3.2)$$

and

- $S_n = \text{Magnitude of slope of control ramp during on-time}$
- $S_f = \text{Magnitude of slope of control ramp during off-time}$
- $S_e = \text{Slope of external ramp}$
For the case where no external ramp is added, \( S_e = 0 \), and \( \alpha = \frac{S_f}{S_n} \). The on- and off-time slopes are equal at a duty cycle of 0.5, and the value of \( \alpha \) is one. At higher duty cycles than 0.5, \( \alpha \) is greater than one. This represents a growing oscillation at the Nyquist frequency, and the current perturbation oscillates about the steady-state condition on alternate switching periods. This is the well-known subharmonic oscillation problem.

Eq. (3.1) also models a constant-frequency control scheme where the clock initiates the \textit{off}-time, and the control voltage initiates the on-time. For this control scheme, \( \alpha \) is given by:

\[
\alpha = \frac{S_n - S_e}{S_f + S_e}
\] (3.3)

This type of control also demonstrates an instability, in this case occurring for duty cycles \textit{less} than 0.5.

The forced response of the constant-frequency controller is shown in Fig. 3.6. The control voltage is perturbed by \( \hat{v}_c \). Notice that the value of the control voltage at time \( t = k \) produces a change in the inductor current at time \( t = k \). (Note: the theory for such systems is derived for perturbations in \( \hat{v}_c \) occurring at any time. However, the discrete-time equation is always derived with the perturbation at time \( t = k \). A standard continuous-time transformation [35] models the continuous-time disturbance.)
Figure 3.6. Constant Frequency Controller with Control Perturbation:

The control voltage $v_c$ is perturbed at time $t=k$, resulting in an inductor current perturbation which is held constant for one cycle.
Combining Eqs. (3.1) and (3.4), the complete discrete-time response is

\[ i_L(k+1) = \frac{1}{R_i} (1+\alpha) v_c(k+1) \]  

(3.4)

The discrete-time equation for the forced response is then given by:

\[ i_L(k+1) = -\alpha i_L(k) + \frac{1}{R_i} (1+\alpha) v_c(k+1) \]  

(3.5)

### 3.3 Continuous-Time Model of Closed-Loop Controller

Computer-controlled systems are used extensively and they have been thoroughly analyzed [35]. A computer-controlled system, which typically consists of an A-D converter, discrete-time algorithm, D-A converter, and the continuous-time system which is being controlled, is shown in Fig. 3.7. Current-mode control has a very similar structure. The control voltage is naturally sampled once in every cycle, and the perturbation in the inductor current is held constant until the next sampling instant. The 'holding' effect of the current is clearly shown in Figs. 3.5-3.6. The discrete-time process, \( H(z) \), is simply the z-transform of Eq. (3.5), given by:

\[ H(z) = i_L(z) = \frac{1}{R_i (1+\alpha)} \frac{z}{z+\alpha} \]  

(3.6)
In order to understand and design the current-mode system properly, we now wish to transform back into the continuous-time domain. Fortunately, current-mode control is just a specific case of the general problem of computer-controlled systems as shown in Figure 3.7, and the transformation is a standard process. The transformation from the z-transform representation of Equation 3.6 to the continuous-time representation of the sample-and-hold system [35] is given by

\[ F(s) = H(e^{sT_s}) \frac{1}{sT_s}(1 - e^{sT_s}) \]  

(3.7)

In other words, we substitute \( e^{sT_s} \) for \( z \) in Eq 3.6, and multiply by the expression \( \frac{1}{sT_s}(1 - e^{sT_s}) \) as explained in [35].

For current-mode control, therefore, the exact, continuous-time, control-voltage-to-inductor-current transfer function, with the current loop closed, is

\[ F(s) = \frac{\hat{i}_L(s)}{v_c(s)} = \frac{1}{R_i} \frac{(1+\alpha)}{sT_s} \frac{e^{sT_s} - 1}{e^{sT_s} + \alpha} \]  

(3.8)

This transfer function now predicts exactly the characteristics of the current-mode cell and it is accurate at frequencies well beyond the switching frequency. However, it is seldom used because of its complexity [28,29]. A new, approximate representation of Eq. (3.8) will be developed in the next chapter, producing very accurate results up to half the switching frequency.
Figure 3.7. Standard Configuration of a Computer-Controlled System: The small-signal representation of the sample-and-hold system is shown, and the analogy with current-mode control is illustrated. The analytical results for computer-controlled systems are directly applicable to current-mode control.
3.4 Continuous-Time Model of Open-Loop Controller

The closed-loop, continuous-time model of the current-mode controller has been found. What is desired, however, is the open-loop model from which the expression for $H_e(s)$ can be found. This can be found from the control-voltage-to-inductor current transfer function found in Eq. (3.8), and from the current-mode cell model of Fig. 3.4.

The modulator gain $F_m$ in Fig. 3.4 is the same as for voltage-mode control. For current-mode control, the ramp is formed by the sensed inductor current, and an external ramp, $S_e$, if constant-frequency control is used, but the modulator gain is still modeled in the same way. The modulator gain for constant-frequency control, with controlled on-time is given by the reciprocal of the height of the ramp that would be obtained if the modulator signal continued with slope $S_n + S_e$ until the end of the cycle. This is the same as the modulator gain for voltage-mode control. The fact that the ramp is derived from the current waveforms is accounted for by feedback loop of the current, and the modulator mechanism is unchanged. The gain is therefore:

$$F_m = \frac{1}{(S_n + S_e)T_s} \quad (3.9)$$

This gain differs from the models [16,18] which had the problem of predicting a current loop crossover in excess of half the switching frequency. (See Chapter 3 Appendix for further discussion of correct choice of modulator gain.)
The modulator gain for constant-frequency control, with the clock initiating the off-time

\[ F_m = \frac{1}{(S_f + S_c)T_s} \]  \hspace{1cm} (3.10)

For constant-frequency control with a naturally-sampled control signal there is no frequency dependence of modulator gain or phase \[36\], and the gain block \( F_c = 1 \).

The duty-cycle-to-inductor-current transfer function can be easily derived from Fig. 3.4 to be

\[ F_i(s) = \frac{i_L(s)}{d(s)} = \frac{V_{ap}}{sL} \]  \hspace{1cm} (3.11)

Recognizing that \( V_{ap} = V_{ac} + V_{cp} \) and that \( S_n = \frac{R_iV_{ac}}{L} \) and \( S_f = \frac{R_iV_{cp}}{L} \), this expression can be rewritten in terms of the control signal slopes:

\[ F_i(s) = \frac{1}{R_i} \frac{S_n + S_f}{s} \]  \hspace{1cm} (3.12)

The product of the modulator gain \( F_m \) and the current gain \( F_i(s) \) is then a single expression for all converters:

\[ F_mF_i(s) = \frac{1}{R_i} \frac{1+\alpha}{sT_s} \]  \hspace{1cm} (3.13)
The open-loop gain term $H_e(s)$ can now be found by equating the closed-loop expression of Eq. 3.8 with the forward gain of Fig. 3.4 divided by one plus the current-loop gain:

\[
\frac{1}{\frac{1 + \alpha}{R_i}} \frac{e^{sT_i} - 1}{sT_s + e^{sT_i} + \alpha} = \frac{F_mF_i(s)}{1 + F_mF_i(s)R_iH_e(s)}
\]

(3.14)

Substituting Eq. 3.13 for $F_mF_i(s)$, the very simple result is obtained

\[
H_e(s) = \frac{sT_s}{e^{sT_i} - 1}
\]

(3.15)

This result was obtained by Brown [29] for the buck converter operating in constant-frequency mode. In this chapter, the analysis is simplified and generalized for all converters containing the current-mode cell.

The expression for $H_e(s)$ has been derived here for the general current-mode control cell, and is not specific to one converter. It will be shown later that it can also be extended to constant on-time and constant off-time control schemes. As mentioned in [29], the form of $H_e(s)$ is difficult to work with when designing a control system. In the next chapter, a simple second-order representation of $H_e(s)$ will be given, capable of accurately predicting current-mode phenomena up to half the switching frequency. This also gives a simple second-order function for the closed-loop, control-to-inductor-current transfer function which will prove to be the most useful for design purposes.
3.5 Discrete-Time Analysis of Open-Loop Controller

The simple form of $H_e(s)$, and the fact that it is the same for all forms of current-mode control and invariant with all power stage parameters except switching frequency, suggest that it could be derived directly with very simple arguments. Consider the modulator function shown in Fig. 3.8, with the control voltage $v_c$ kept constant. A perturbation $i_L(k)$ is introduced at time $t = k$. This perturbation affects the next switching instant, causing a perturbation in the switching time $t(k + 1)$ where

$$\hat{i}_L(k) = -(S_n + S_e)\hat{t}(k + 1)$$ (3.16)

for constant-frequency control, with a clock initiating the on-time. The perturbation in duty cycle is then related to the perturbed current by

$$\hat{i}_L(k) = -\frac{1}{F_m}\hat{d}(k + 1)$$ (3.17)

Eq. (3.17) is intentionally written in this form, with the duty cycle on the right-hand side, and with the change in current at time $t = k$ dependent upon the duty cycle at time $t = k + 1$. It has been shown that the variable $\hat{i}_L(k)$ is the 'held' variable of the system, remaining constant from time $t = k$ to time $t = k + 1$. The duty cycle $\hat{d}$ is the sampled input, being just a scalar multiple of the control voltage, $v_c$, and the current feedback signal, $i_L$. The correct z-transform representation of this sampled-data subsystem is then given by the equation
Figure 3.8. Current-Mode Control Modulator with Perturbation in Current:

The control voltage $v_c$ is constant, and a perturbation in $i_L$ is introduced at time $t = k$, causing subsequent change in duty cycle $d(k+1)$. 
\[ \hat{i}_L(z) = -\frac{1}{F_m} z \hat{d}(z) \]  

(3.18)

The corresponding sampled continuous-time model is

\[ \hat{i}_L(s) = -\frac{1}{F_m} e^{sT_i} \frac{1-e^{-sT_i}}{sT_s} \hat{d}(s) \]

(3.19)

\[ = -\frac{1}{F_m} \frac{e^{sT_i} - 1}{sT_s} \hat{d}(s) \]

The gain block \( H_e(s) \) from Fig. 3.4 and from Eq. (3.19) is then

\[ H_e(s) = -\frac{1}{F_m} \frac{\hat{d}(s)}{\hat{i}_L(s)} \]

(3.20)

\[ = \frac{sT_s}{e^{sT_i} - 1} \]

Notice that gain terms containing \( R_i \) appear in the expression for the current transfer function, and the modulator gain, and these terms are cancelled. This is the same result that was obtained indirectly in the previous part of this chapter, confirming the analysis approach.
3.6 Extension of Modeling for Constant On-Time or Constant Off-Time Control

Constant on-time or constant off-time modulators are sometimes used instead of constant-frequency modulators [37]. These control schemes can offer advantages of low audio susceptibility, ease of implementation, or lower power stage weight. Also, converters with this modulation do not exhibit the subharmonic oscillation problem associated with constant-frequency control. The most commonly-used scheme is the constant off-time modulator, since the inductor current is easily sensed with a current transformer in series with the power switch.

There are many different schemes which can be used to provide constant on- or off-time modulation. Fig. 3.9 shows the most commonly-used scheme for generating constant off-time control. A PWM ramp signal (the inductor current signal for current-mode control) is started when the power switch is turned on, and this provides the turn-off command when the ramp intersects the modulated control signal, $v_c$. This scheme gives an on-time pulse of varying width. The falling edge of the PWM output pulse starts a timer which provides the fixed off-time signal. The falling edge of the timer triggers the turn-on of the power switch. An external ramp is rarely used for constant off-time modulation, and it is not considered here. The constant on-time controller is the dual control scheme of constant off-time. A timer provides a fixed on-time, and the current ramp, or PWM ramp,
is used to turn the switch on. In this case, the valley of the inductor current not the peak of the current, is used to control the turn on.

It is easy to show that the average gain of the constant off-time modulator is given by

\[ F_m = \frac{D'}{S_n T_s} \]  

(3.21)

Notice that this is the same gain as that for the constant-frequency modulator, multiplied by \(D'\).

The constant off-time modulator described above shows an interesting characteristic. Unlike the naturally-sampled constant-frequency modulator which has flat gain and phase [36], the constant off-time modulator exhibits a phase-lead characteristic which increases linearly with duty cycle and with frequency. Fig. 3.10 shows the measured phase characteristics of the constant off-time modulator for different duty cycles, and compares this with the constant-frequency modulator with a duty cycle of 0.45.

It is important to emphasize that the exact mechanism for the generation of the constant off-time pulse is critical in predicting the small-signal characteristics of the modulator. There are many different timing mechanisms for the generation of this signal, and a few of these are described in [8,9]. It is also shown in these references that one interesting modulator actually can generate ninety degrees of
Figure 3.9. Constant Off-Time Modulator Waveforms:

A timer is started when the switch is turned off to provide a constant off-time pulse. A ramp is initiated when the switch is turned on, and the intersection of the ramp with the modulation signal is used to provide the signal to turn the switch off.
phase lead at all frequencies. The phase lead in general for these schemes is produced by an effective differentiation of the input signal by the modulator.

The constant off-time modulator with a duty cycle of only 0.1 gives a small phase lead of only 10 degrees at half the switching frequency. However, this phase lead increases to 45 degrees at a duty cycle of 0.5, and to 90 degrees at a duty cycle of 1. The gain of the modulator, not shown in Fig. 3.10, also shows an increase at frequencies approaching half the switching frequency at higher duty cycles, but this effect is not modeled. This gain change is produced by sidebands of information appearing at higher duty cycles. Exact analysis of this effect [8,9] is possible, but any incremental enhancement to the accuracy of the new current-mode model is insignificant.

For constant-frequency control, the gain block, $F_c$, of Fig. 3.4 was unity since there was no frequency dependence in the modulator gain. For variable-frequency control, this gain term is used to account for the phase dependency of the modulator, and it will be shown that the model of Fig. 3.4 is equally valid for variable frequency control with the appropriate values of $F_c$. For constant off-time, it is given by

$$F_c = e^{sDT_c/2} \quad (3.22)$$

For constant on-time it is given by

$$F_c = e^{sDT_c/2} \quad (3.23)$$
Figure 3.10. **Constant Off-Time Modulator Phase Measurement:**

The constant off-time modulator shows a linear phase lead, the slope of which depends on the duty cycle. The constant on-time modulator also shows a phase lead, the slope of which depends upon the complement of the duty cycle, \( D' = 1 - D \).
These terms are approximate: at frequencies close to the switching rate, an increase in gain is observed. However, these approximations are adequate for purposes of the current-mode control model. Notice that the product of the gains $F_c F_m$ gives the complete modulator gain for voltage-mode control.

The model for constant on-time and constant off-time control is best derived by comparing the system response with that of constant-frequency control with an appropriate external ramp. Fig. 3.11a shows the control waveforms for a constant-frequency converter with an external ramp equal to the off-time slope of the sensed current waveform. A perturbation in the current at time $t = k$ is exactly damped out in one switching cycle. After the next sampling instant, the inductor current returns to its previous trajectory. This system corresponds to a 'dead-beat' control scheme. The expressions of Eqs. (3.1) and (3.5) are valid with a value of $\alpha = 0$. Referring to Eq. (3.6), the z-transform equation then has a pole at the origin which corresponds to dead-beat control. The modulator gain of the constant-frequency control, given in Eq. (3.9), for an external ramp equal to the off-slope of the current signal, reduces to:

$$F_m = \frac{D'}{S_n T_s} \quad (3.24)$$

This is the same modulator gain as that for the constant off-time control, given in Eq. (3.21).
Figure 3.11. **Comparison of Constant-Frequency and Constant Off-Time Control:**

The natural response of the current-mode controller for constant frequency with external ramp equal to the off slope is shown with constant off-time control. Both systems exhibit ‘dead-beat’ control characteristics.
Fig. 3.11b shows the natural response of the constant off-time controller, with no external ramp. Like the constant-frequency controller described above, the perturbation from steady-state is exactly damped out after the next sampling instant. The constant off-time modulator gain, given in Eq. (3.21) is the same as that for the constant-frequency control with the external ramp. It is important to note that the constant off-time system perturbations are compared to the steady-state waveforms shifted in time to correspond to the new frequency. It was shown in [38] that the small time shift is a second-order effect which can be ignored. The current-loop dynamics of the constant off-time (and constant on-time) system are the same as those for the constant-frequency control with the external ramp equal to the off-time slope. The model of the current loop in Fig. 3.4 is valid for each of these forms of current-mode control, with the same invariant value of the sampling gain, $H_e(s)$, as was found previously. The phase lead term, $F_c$, does not appear inside the current loop gain.

An important difference exists for constant on-time and constant off-time controls for the control-to-output response. Notice that the model of Fig. 3.4 contains the gain block $F_c(s)$ which introduces a phase lead in the system. In the absence of current feedback ($R_i = 0$), the forward gain of the converter from control voltage to the output voltage is simply the modulator gain for voltage-mode control, $F_c F_m$, times the power stage gain from duty cycle to output voltage. The control-to-output characteristic for current-mode control must also show this phase-lead characteristic for the model of Fig. 3.4 to be correct for variable-frequency control.
Fig. 3.12a shows the response of the constant-frequency control, with $S_c = S_f$, with a control input perturbation, over several cycles of operation. The waveforms of the small-signal perturbations show the inductor current signal changing in discrete steps (if the transition slopes are ignored), and tracking the control signal exactly. Again, this is a feature of dead-beat control systems. Fig. 3.12b shows the response of the constant off-time converter to the same input perturbation. In this case, the small-signal perturbation is slightly different. Due to the nature of the control, the perturbations in current appear as discrete pulses that terminate before the end of the switching cycle. It is important to note that the average magnitude of the perturbation over the full cycle is equal to the perturbation of the constant-frequency control system.

The variations in the small-signal perturbations provide a phase lead relative to that of the constant-frequency system. The width of the pulse with the constant off-time control is given by

$$P_w = D'T_s$$

(3.25)

At a duty cycle of zero, the pulse width is equal to that of the constant frequency control system, and there is no phase lead. This situation is shown in Fig. 3.13a. At close to unity duty cycle, the pulse becomes an impulse function, and the information carried leads that of constant-frequency system by half the switching period. At half the switching frequency, this corresponds to a 90 degree phase lead, which is predicted by Eq. (3.22).
Figure 3.12. Comparison of Constant-Frequency and Constant Off-Time Control:
The forced response of the current-mode controller for constant frequency with external ramp equal to the off slope is shown with constant off-time control. The off-time response carries the same average information in shorter pulses of current perturbations.
Figure 3.13. **Constant Off-Time Responses at Different Duty Cycles:** At duty cycles close to zero, the response of the constant-frequency and constant off-time controls are practically identical. At duty cycles close to one, almost 90 degrees phase lead occurs at half the switching frequency with constant off-time control, due to the shorter pulses of information.
Fig. 3.14 show the perturbation waveforms of Fig. 3.13 more clearly, together with the modulation information carried by the pulses. The shorter pulses at duty cycles close to unity advance the information relative to that of the short duty-cycle system, by half the switching period. An alternative way of looking at this is that the duty cycle close to unity samples the input modulation signal with impulse functions (or an approximation thereof). Impulse sampling does not introduce a phase delay. However, the step-function sampling obtained with the duty-cycle close to zero produces a phase delay equal to half the switching period, as is well known from communication theory [8,9].

The constant off-time control scheme, therefore, shows exactly the same response as the constant-frequency control scheme, when an external ramp equal to the off-time slope is used, with the exception of a phase-lead of the inductor current information, predicted by the value of $F_c$ given in Eqs. (3.22-3.23). The constant off-time and constant on-time current-mode schemes, therefore, are accurately modeled by the small-signal model of Fig. 3.4.
Figure 3.14. Modulation Information Carried by Constant Off-Time Modulator:

At duty cycles close to unity, modulation-frequency information carried by the perturbations in the inductor current is time-shifted by almost half the switching period relative to that carried by the modulator at close to zero duty cycle.
3.7 Conclusions

Several important results were derived in this chapter for the four common forms of current-mode control. Firstly, a generic current-mode cell was derived for all PWM converters using current-mode control. The discrete-time response of the closed-loop control-to-inductor-current for constant-frequency control of the current-mode cell was found to be

\[
\hat{i}_L(k+1) = -\alpha \hat{i}_L(k) + \frac{1}{R_i} (1+\alpha) \hat{v}_c(k+1) \tag{3.26}
\]

The continuous-time representation of the system, derived from standard results for sample-and-hold systems, is

\[
F(s) = \frac{i_p}{v_c}(s) = \frac{1}{R_i} \frac{(1+\alpha)}{sT_s} \frac{e^{sT_s} - 1}{e^{sT_s} + \alpha} \tag{3.27}
\]

This equation was used to derive the open-loop gain block \(H_e(s)\) shown in Fig. 3.4. This block was found to have a common form for all converters and all four forms of current-mode control discussed in this dissertation:

\[
H_e(s) = \frac{sT_s}{e^{sT_s} - 1} \tag{3.28}
\]

This high-frequency gain block is sufficient to characterize the crucial characteristics of current-mode control with constant frequency operation. Notice that the correct form of \(H_e(s)\) can only be found if the proper version of the modulator gain is used.
For constant on-time and off-time control schemes, it was found that a phase-dependent term, $F_c$, added to the model of Fig. 3.4, provides a general model for all control schemes for the current-mode cell. (The gain $F_c$ is unity for constant-frequency control.)
When performance counts . . .

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