The complexities of power electronics systems were encountered very early in their history. Systems integration of multiple converters and power sources caused problems with system stability. The systems built were too large and complex for brute-force analysis or simulation. Fortunately, Dr. Middlebrook of Caltech was working on theories to simplify large system analysis, and this was applied very effectively to the problems being encountered in shipboard and aircraft power.

The tools and techniques applied over 25 years ago are still of paramount importance to designers today. In this article, we'll review the most important aspects of this early work regarding input filter modeling, and show how you can quickly apply the results to your system design.

A submarine control console shows the pervasiveness of electronics.

These systems rapidly embraced the use of switch-mode power supplies, and system interaction issues drove early power supply analysis.
Early users of switching power systems were military and space applications. The great improvements in density and efficiency of switching power supplies were crucial to confined systems—those with limited space and power-generating capabilities. Shipboard and airborne power depend heavily on switch-mode power supplies. The specific needs involved drove much of the early analysis and development of power supply theory and research.

Over two decades ago, the US Navy and Air Force encountered a problem in the application of switching power supplies. Jerry Foutz, a trusted advisor to Navy Program Managers, delved into the problem. They worked fine on the test bench, yet when placed in the system, they oscillated. These problems led to seminal power supply design analysis by Dr. Middlebrook of Caltech.
Jerry Foutz has been involved in power electronics systems for over 25 years, and initiated early research work into filters that is so important to us today. Early guidelines used for EMI filter design were found to be flawed when applied to switching converters, as Mr. Foutz and others were discovering early in power electronics history. He describes how events unfolded:

“Familiar with the 1971 Yu and Biess papers and Nathan Sokal’s 1973 paper, I set out to make a simulation using CSMP software on a PDP11-40 computer of how adding an EMI filter to a switching-mode power supply caused the combination to go unstable. This was in 1974 at the Naval Ocean Systems Center (NOSC).

“What I found was the published criteria didn’t really work. The criteria predicted oscillations in stable systems and did not predict oscillations in unstable systems.

“At Powercon I (Beverly Hills, March 20-22, 1975), I brought up the question from the floor to a panel of chief engineers of power supply companies. None of them had heard of the problem and generally did not believe it. However, several others in the audience had experienced it and we met at break time. There was enough experience in this group to report after the break that it was a real problem and should be considered by designers.

“Later, I tried to get Thomas Wilson at Duke University to look at the problem, but Duke was swamped with NASA work. What I wanted were two procedures for MIL-HDBK-241. One procedure would let a designer design an EMI filter and switching-mode power supply combination that would not oscillate. The other procedure would allow a filter to be designed having only "black box" measurements on the power supply.

“At PESC’75 (Culver City, June 9-11, 1975) I talked to R. David Middlebrook at Caltech, who had just given a paper in which he discussed the problem. He thought he could do what I asked using a new canonical model of switching-mode power supplies developed at Caltech. I funded the work. The result was both a section for MIL-HDBK-241 and Middlebrook’s landmark IEEE paper.

“After Middlebrook completed his paper for a single power supply and filter or source, the question came up - what happens if many power supplies are added onto an aircraft or shipboard generation, whose simple model is a series generator, resistor, and inductor?

At the time, switching power supplies were a small part of load of ships and aircraft and no one knew. I asked Middlebrook if he could look at the problem, and he would if a shipboard or aircraft with loads could be made available to work both theory and measurements together. I could not make this happen.

“The shipboard people at the responsible lab mostly ignored the problem, but the aircraft lab people were very concerned because the were pushing 270 VDC generators for aircraft to work with switching-mode power supplies, including filters that could cause the whole system to go unstable. They solved the problem by making switching-mode power supply simulated loads, including filters, for testing the aircraft generators and power system. Later the shipboard people followed in this approach- all empirical, nothing theoretical.

“The submarine anecdote was something else, but related to Middlebrook’s philosophy of starting with a simple model and then only adding, verifying by measurement, until it meets your design needs.

“The Navy lab responsible for developing computer models of shipboard systems took the approach of making the most complex computer model of each component they could and then, if needed to get it to simulate in a system, simplifying it until you could get it to run in the system. The problem was that the models were so complicated, no one could get the parameters to run it unless they had the actual component to make measurements on and then it was almost impossible because some of the parameters were internal and not accessible to direct measurement. Also, no one knew how to simplify the models. To the best of my knowledge, they were never able to simulate an actual ship’s system with this approach.

“Another lab, who was responsible for results for submarines only, took the opposite approach of using very simple models that would allow the simulation of a submarine electrical system and loads in less than 100 lines of Fortran code including filter effects. It gave them the answers they needed to tweak the system in almost real time and was as accurate as the measurements they took to find and solve problems on actual submarines.

“This is just one of many real-life anecdotes I could spin on superiority of Middlebrook’s modeling philosophy, which, as I understand it and describe it is to start with the simplest model possible and only adding complexity to it as needed to get it to meet your design needs, using measurement to decide if the model is good enough for your needs.”

Additional information may be found on Jerry Foutz’s website: www.smpstech.com
Dr. David Middlebrook’s Story

The Caltech Power Electronics Group (PEG) began the pioneering work on circuit averaging, needed for the complex nonlinearity of switch-mode power supplies. Dr. Middlebrook remembers how the Navy needed help with their problem:

“When Jerry brought the Navy’s problem to my attention, I immediately realized that it could be treated in general, for any converter, by use of the canonical model based on state-space averaging, which we had just completed but not presented until PESC ’76. So that’s how it happened.

As I emphasized in the resulting “input filter” paper, the important question is not merely how to design the input filter to avoid instability, but how to design it so that the properties of the converter are not significantly disturbed.

A bit of insight that isn’t mentioned anywhere else: the analysis that led to the “inequality criteria” was done by direct application of the Extra Element Theorem (EET), but because the EET was unpublished, I couldn’t mention it! In fact, the first EET paper wasn’t published until 1989; now, in my courses, I use the input filter problem as an example of how useful the EET is in design-oriented analysis, and I usually tell this story.

Jerry provided contract support to the Caltech PEG from the Navy, and later from Rockwell. I remember we used to have “Rockwell days” at Caltech, when a contingent of Rockwell engineers came to hear us present our latest work. On one of those occasions, I remember summarizing the later work by others showing that the inequality criteria are essentially the same for a current-programmed converter.”

The extra-element theorem (EET) is a great way of simplifying the analysis of an otherwise intractable system. Unless you’ve been to one of Dr. Middlebrook’s courses, this theorem may sound foreign. Fortunately, there are two new books on the subject soon to be published. The first, by Dr. Middlebrook himself, is eagerly anticipated by all who know his work.

The second is from Dr. Vatche Vorperian, who describes further applications of analysis simplification techniques. Vorperian’s book devotes a lot of space to power supply analysis. Dr. Vorperian’s work is invaluable in modeling, and we rely heavily upon it in our design software. He is, to date, the only person to have tackled many of the complex analytical converter equations and succeeded, including all parasitics of components.

In modern switch-mode power supply design, we are taught the basics of analysis for the fundamental converters. Life gets complex enough for just these simple building blocks, with voltage-mode control, current-mode control, multiple outputs, and many other issues we have to deal with as designers.

Switching power supplies are now finding their way into almost every industry. But they don’t come in the simple packages that you might study in a power electronics class. They are imbedded in other circuitry with protection, filters, batteries, loads, and many other converters. Yet every active switching power supply has a compensated feedback loop to regulate the output, and every power supply in the system has the potential to cause instability.

For large power systems, such as the International Space Station there are literally hundreds of power converters. The configuration of the system can change as the mission of the space station is defined over the years. And there is a potential for all of the power converters to interact with each other. This has led to the stringent specification of power converters for such systems. Sometimes this can result in overly conservative design, but it is better to be safe than sorry for such expensive systems.

Early on in power electronics, problems were encountered when connecting an input filter to a switching power supply. This phenomenon has been thoroughly analyzed in many papers, and is well understood. The “Middlebrook” criterion is applied to make sure that a switching power supply will not become unstable when an input filter is added.
Who has a Problem?

We've shown in earlier issues of *SPM* how involved the analysis of a simple power supply can be, especially when current-mode control is used. Adding an input filter, as the Navy discovered, can make something go awry. The system is immediately very complex with the number of state variables introduced. Other large systems such as the IBM mainframe in our July 2000 issue, and the International Space Station in our January 2001 issue, are obvious examples where we might expect complications.

But even small power users create complex systems. It's quite common to place many small converters on a board together, all feeding different (or the same) load, and fed from a common input voltage with input filters. All of these systems are prone to system stability issues.

In short, every switching power supply built has the potential for input filter oscillation problems. A power supply should never be built without an EMI filter, even if you use very simple converters.

Simple Filter Explanation

Fig. 1a shows a simple LC input filter, with a damping resistor across the capacitor. In reality, the damping is much more complex than this, but the simple example explains what happens. The input filter will act like a two-pole system. If underdamped, it will have a tendency to ring, and the oscillations will be damped according to the value of the resistor, R.

Before switching regulators came along, the EMI filter fed a linear regulator. A linear regulator has the input impedance characteristic of a current source. Whatever happens to the input voltage, the current draw is constant. With an ideal linear regulator with high bandwidth, the input filter damping is unaffected by the connection to the regulator.

Switching regulators were immediately found to be more problematic. The power supply DC input characteristics are shown in Fig. 1b. This is a constant power curve for a 1000 W supply operating at 85% efficiency. The DC input resistance, at any given point, is determined by the slope of the I-V curve. The asymptote drawn in the figure at 270 V input gives the lowest value of the power supply input resistance. As the voltage is raised, the input resistance also climbs. We are concerned about interaction with the lowest value of input impedance, so analysis is done at this point.

The important thing to note about the input resistance is that it is *negative*. An increase in input voltage causes a decrease in input current. As you may remember from early circuit theory and electronics classes, negative resistance, usually formed by simple active devices, is what we use to build oscillators.

When the power supply is connected to the input filter as shown in Fig. 1c, the negative input resistance of the converter combines with the positive damping resistance of the filter, resulting in complete elimination of any damping.

The previously damped filter will now ring indefinitely with any perturbation.

If the input resistance of the regulator has a *higher* value than the damping resistor, some positive resistance will remain, and the system will still be damped (but less than without the regulator.)

If the input resistance of the regulator has a *lower* value than the damping resistor, the resulting combined resistor is negative, and the system will oscillate.
This is the basis of the Middlebrook impedance criterion, applied to a simple resistive case.

**The Generalized Impedance Criteria**

Of course, in most converters you will not see a filter as simple as that described above, and the power converter certainly is not ideal. So, we use a generalized statement of the simple resistive interaction, as shown in Fig. 2.

The way we usually apply the impedance criteria is paraphrased as follows:

> If the input impedance of the power supply is much greater than the output impedance of the input filter, there will be no problems with stability of the power system.

There is more to it than that, of course, as explained in the original papers. This simple rule, however, is sufficient to apply to your converter design.

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**Which input impedance should you use?**

There are three flavors of input impedance of a switching regulator. The first, and simplest, is the calculated negative input resistance obtained from the DC characteristics, as in Fig. 1b.

A more complex representation of this is the true closed-loop input impedance, considering the finite control loop bandwidth, and all of the real components of a power supply.

The third input impedance to consider is the open-loop input impedance of the power supply, before the feedback control loop is closed.

Fig. 3a shows an example 1 kW full-bridge power supply that we will use throughout this article. Voltage-mode control is used for the converter, and the control loop is closed with a crossover frequency of about 7 kHz at low line input. The three different input impedances are plotted in Fig. 3b.

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**Side Note:** Input EMI filters contain both common-mode and differential mode elements. The common-mode elements do NOT come into play in the filter interaction issue. They must, of course, be properly damped to avoid ringing with respect to chassis ground, but you don’t have to worry about the negative resistance effect on the common mode damping.
The open-loop input impedance shows the resonant dip of the LC filter. With a very underdamped system, desirable for a high-efficiency switching converter, this dip in impedance can be very sharp with a very low minimum value.

The control loop of the converter eliminates the LC filter resonant dip, and at low frequencies, it will exactly track the predicted input resistance. At higher frequencies, where the control loop is no longer effective, the input impedance rises, with the asymptote determined by the inductor of the power stage.

The final curve plotted shows the fixed resistance calculated for the converter. As we will see later, this is the curve that is usually used in industry for a variety of practical reasons.

Now let's look at different design cases of an input filter, and see how to apply the design rules.

**Design Case 1:**

**Z out << Open Loop Z in**

In Middlebrook's early paper, he talked about how to avoid any interaction at all in a power supply by looking at the open loop input impedance of a converter, and making sure that the output impedance of the filter is always less than this quantity.

Such a case is shown in Fig. 4. This input filter has a large bulk capacitor that damps the resonance and keeps the output impedance low. For off-line power supplies, this capacitor is commonplace since it is needed for line-frequency energy storage.

The impedance curves shows that there is no overlap in this case. And the control-to-output transfer function, measured from the output of the error amplifier to the output of the power supply shows just a small perturbation from the power system without an input filter, confirming Middlebrook's results.

This is the ideal design case that completely separates the building blocks of a power system. Unfortunately, we can rarely arrive at this design. If you have full control of both the power converter design and input filter design it can be done, but you will soon find that it is overly constraining on your choice of filter components. The resonant dip of the input impedance can force very large values of filter capacitor which may not be practical.

![Design Case 2](image-url)
**Z out << Closed Loop Z in**

Fig. 5 shows a case where the filter output impedance is less than the closed-loop input impedance, but greater than the open-loop input impedance. Now we can see a severe perturbation in the control-to-output transfer function with a sharp dip in gain where the impedance cross each other, and a sharp bump in phase. The change in the input filter is a reduction of the large capacitor by a factor of 10.

For the mathematically inclined, this “glitch” in the control transfer function is caused by a pair of complex zeros placed very close in the s-plane to a pair of complex poles caused by the addition of the input filter. The near-cancellation of the poles and zeros cause the control transfer function to quickly return to where it was before the addition of the input filter.

The power supply is not unstable. The control glitch may be a little unsettling, but this is common design procedure. You should not be concerned about this.

That's not to say, however, that this is a good filter design. While the system is stable, the input filter is not well damped. Perturbations in the system will cause damped ringing at the interface between the filter and the power supply. A good design will damp this ringing properly, but it is not necessary for the resulting output impedance to be lower than the open loop input impedance.

Designing to avoid the closed-loop, rather than the open-loop input impedance is the norm for practicing design engineers. I asked Dr. Vorperian how he applies the Middlebrook criteria. He replied, “I do it the simple way - I assume the power supply has infinite bandwidth, and negative input resistance. Just damp against that.”

**Design Case 3:**

**Z out > Closed Loop Z in**

Fig. 6 shows another filter design where the inductor is now increased to 2 mH. The output impedance now exceeds the closed loop input impedance, and the system is in trouble. The control-to-output phase occurring at the control glitch now drops down by an additional 360 degrees, and the system will oscillate.

Again, for those who like math, the open-loop system has an additional pair of complex poles from the addition of the input filter, and a pair of right-hand-plane zeros at almost the same frequency. This causes the gain to return to where it was without the input filter at higher frequencies, but the phase drops 180 degrees rather than recovering 180 degrees.

We know these are right-half plane zeros and not right-half plane poles, since we have not yet closed any active control loop. It is not possible to move any poles to the unstable part of the s-plane. There’s not much hope of stabilizing this system with any easy control—or is there?
Complex zero pair is now in the RHP causing additional 360 deg phase delay.

**Fig. 6: Input filter design that oscillates**
Design Case 4: The “Magic” of Feedforward

Fig. 7 shows a control system that magically seems to fix the problem of input filter interaction. The ramp of the PWM controller is now derived from the input voltage to the power stage, with its height proportional to the voltage. As the input voltage goes up, the ramp slope increases, and the duty cycle of the converter automatically and immediately cuts back.

This is known as “feedforward” control, and there are several PWM control chips that can be used to implement this control scheme. They have their advantages, which we won’t go into in this article.

A feedforward loop is shown applied to the circuit of Fig. 7. The control transfer function for this case shows that the glitch has completely disappeared, even though we are violating the impedance criteria. So is the system stable? It certainly looks stable, yet it is not. It doesn’t require detailed math to show why it is not stable, just a little bit of control theory.

The term feedforward has a very specific meaning to control theorists. It is created by sensing inputs to a control system, where an input is defined as either an independent voltage or current source for electrical circuits. And when a feedforward loop is created, it has the effect of moving the zeros of a transfer function, not the poles. For the system above, this would be a desirable effect, since the zeros were in the right-hand plane.

But when the “feedforward” control scheme is applied to the circuit as shown, it is no longer feedforward in the control sense. The sensed quantity is no longer a voltage source. It is now the output voltage of the filter, formed by a combination of the state variables of the system. To control theorists, if you sense state variables and use them for control, this is feedback, regardless of where in the system they are sensed. And feedback can only move pole locations, not zero locations.

So how did the transfer function get fixed? The controller places a pair of RHP poles exactly on top of the RHP zeros of the previous example, so you can no longer see any perturbation in this transfer function. In control theory, what we have built is an unobservable system. The transfer function being plotted cannot show what is happening internally to the converter.

This is a case where loop gain measurement, and applying the Bode criteria doesn’t work. That doesn’t mean that theory is violated. Bode criteria were never meant to be applied to non-minimum phase systems, so the Nyquist criteria must be applied. Nyquist involved encirclement of the -1 point in the s-plane, the same number of times as there are RHP poles in the system. That’s fine in theory, but of course you must first know that the RHP poles are there.

The bottom line is that feedforward control doesn’t modify the input filter design rules at all. Check the impedances, and if they violate the stability criteria, redesign the filter regardless of the appearance of your control transfer functions.
Design Case 5: Current-Mode Control

Feedforward control may be something you have never used, anyway, and don't plan to use. But that doesn't mean you won't run into trouble. The same effect happens with current-mode control, which creates an inherent feedforward path.

As shown in Fig. 8, the control transfer function with current-mode looks fine, with just a small glitch. (Cancellation is not perfect in this example as it was for feedforward control.) But the input filter design criteria is violated, and the system will become unstable. Again, don't just look at the control transfer functions, they can be misleading. You should look at the input filter impedances, just as you did for voltage-mode control.

Defining the “Filter” and Power Supply

When buying a prepackaged power supply, or brick, it will always include some filtering components, even if just a small high-frequency bypass capacitor. Module vendors may not name their filter and power supplies by the name you rely on for impedance criterion.

This is shown in Fig. 9a. The module with the power supply and internal filter components is called the “power supply” and the separate filter is called the EMI filter. This is not where you want to apply the impedance criterion. The internal filter components in the power supply module need to be combined with the input filter, and the impedance compared at the point shown in Fig. 9b.

If you are doing system modeling on purchased supplies, this can present practical problems. Some power supply vendors think that their filter designs are top-secret, and won't tell you what is inside the module. This situation usually leads to an overdesigned filter. The input capacitor in the power supply lowers the input impedance, and the output impedance of the filter must also be lowered to avoid interaction with the unknown system. You should always try to get as much data on the input filtering inside the modules as possible to design the best system.

The proper definition of “where the filter ends and the power supply begins” is crucial for minimizing input filter size. If you are not looking at the proper point, the impedance criteria for stability is no longer valid. It only applies when you are looking into the negative impedance point of the switching converter cell.

In fact, violating the impedance criteria at a point such as that shown in Fig. 9a can actually lead to a more stable system. A large input capacitor inside the converter module is a good component to have. It reduces the Q of the input filter, which properly defined, includes the input capacitor. However, it reduces the input impedance of the converter module significantly, and makes interaction with the rest of the filter likely.

This is a problem that designers of large aerospace systems face. Input impedance is specified for the converter including the EMI filter. This rules out the use of large input capacitors, and requires a more complex input filter design.
In the Space Station article in January 2001 SPM, we discussed how converter designers had to meet minimum input impedance requirements even when a short circuit was placed on the output of the supply. This can be a very difficult technical challenge.

Making Measurements

Finally, it is necessary to talk about the method of measuring impedance quantities. Aerospace designers are frequently required to provide measurements of the input impedance of their supplies over all conditions of line and load to ensure that they will not present a load that is too heavy on the distribution bus.

Fig. 10a shows a method for injecting a signal into the high-voltage interface between the input filter and the power supply. A power FET is connected as a source follower with a high impedance dc connection to the drain (resistor value will depend on the device used and the input voltage on the bus.) An isolated AC signal from the network analyzer is connected to the gate.

One probe of the network analyzer is connected to one input of the power supply, and a current transducer (resistor or current probe) to the other input. The ratio of these two measurements gives the input impedance.

The input impedance measurement setup is not particularly convenient to construct. The FET pass device must be heatsinked for higher power supplies, and the network analyzer must be interfaced with high voltages, sometimes referenced to an AC line, not ground.

The use of differential isolation probes can help with this issue, and is recommended to protect your measurement equipment. And once you are done with the measurement, you still don't have the quantity needed. Unless an end customer requires it, this is not a recommended measurement to make. Just calculate the negative input resistance, and use that number.

Note: for the setup shown, the quantity being measured in the power converter input impedance in parallel with the input capacitors. The true input impedance of just the convert cell must be extracted from the measurements and the known impedance of the capacitors. It is not practical to measure the input impedance of just the switching cell directly - the measurement process would be too invasive in the high-frequency pulsating current section of the circuit.
The output impedance of the filter, however, is an easy measurement to make. Move all of the filter components to the left of the measurement point to make sure that you measure the proper filter. It is not necessary to power up the system to make this measurement, but you must short circuit the input of the filter in order to get the proper measurement. If there is a long cable run from the input of the power system to the power source, you should include this cable in your measurement. Cables, isolation transformers, and variacs, have all been known to raise the impedance of an input filter to the point where they cause a power system to become unstable.

References:

There are numerous, perhaps hundreds, of papers on input filter design and interactions. You can find many of these at Jerry Fouat’s web site if you have an interest in power electronics history.

www.smpstech.com